DESCRIBING FUNCTIONS 1.1.3 through 1.2.2

In addition to introducing students to the classroom norms of problem-based learning, the main objective of these lessons is for students to be able to fully describe the key elements of the graph of a function. To fully describe the graph of a function, students should respond to these graph investigation questions:

Graph Investigation Question	Sample Summary Statement
What does the graph look like?	*The graph looks like half of a parabola on its side.*
Is the graph increasing or decreasing (reading left to right)?	*As x gets bigger, y gets bigger.*
What are the *x*- and *y*-intercepts?	*The graph touches both the x- and y-axes at 0.*
Are there any limitations on the inputs (domain) of the equation?	*Only positive values of x are possible. Zero is also possible.*
Are there any limitations on the outputs (range) of the equation? (Is there a maximum or minimum *y*-value?)	*The smallest y-value is 0. There is no maximum y-value.*
Are there any special points?	*The graph has a "starting" point at (0,0).*
Does the graph have any symmetry? If so, where?	*This graph has no symmetry.*

The more formal concepts of function and domain and range are addressed in Lessons 1.2.4 and 1.2.5.

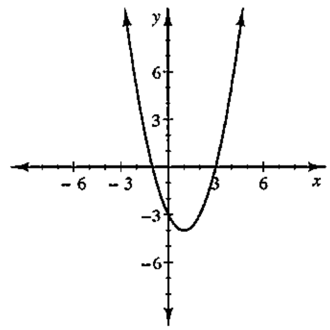
For more information, see the Math Notes boxes in Lessons 1.1.1, 1.1.2, and 1.1.3. Student responses to the Learning Log in Lesson 1.1.1 (problem 1-32), if it was assigned, can also be helpful.

Example 1

For the equation $y = x^2 - 2x - 3$, make an $x \rightarrow y$ table, draw a graph, and fully describe the features.

At this point there is no way to know how many points are sufficient for the $x \rightarrow y$ table. Add more points as necessary until you are convinced of shape and location.

x	–2	–1	0	1	2	3
y	5	0	–3	–4	–3	0



Be careful with substitution and Order of Operations when calculating values. For example if $x = -2$,
$y = (-2)^2 - 2(-2) - 3 = 5 = 4 + 4 - 3 = 5$.

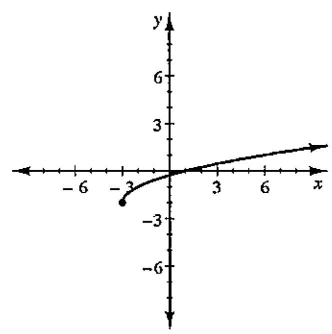
The graph is a parabola; it points upward. The x-intercepts are $(-1, 0)$ and $(3, 0)$. The y-intercept is $(0, -3)$. Reading from left to right, the graph decreases until $x = 1$ and then increases. The minimum (lowest) point on the graph (called the vertex) is $(1, -4)$. The vertical line $x = 1$ is a line of symmetry. There are no limitations on inputs to the function. Outputs can be any value greater than or equal to –4.

Example 2

For the equation $y = \sqrt{x+3} - 2$, make an $x \rightarrow y$ table, draw a graph, and describe the features.

Note that the smallest possible number for the $x \rightarrow y$ table is $x = -3$. Anything smaller will require the square root of a negative, which is not a real number.

x	–4	–3	0	1	3	6
y		–2	–0.3	0	0.4	1



The graph is half a parabola. It starts at (–3, –2) and has x-intercept of (1, 0) and y-intercept of $(0, \approx -0.3)$. The graph increases from left to right. The inputs are limited to values –3 or greater, and the outputs are limited to –2 or greater. There is no line of symmetry.

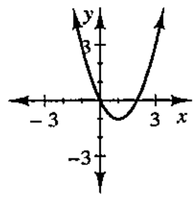
Problems

For each equation, make an $x \rightarrow y$ table, draw a graph, and describe the features.

1.	$y = x^2 - 2x$	2.	$y = x^2 + 2x - 3$	3.	$y = \sqrt{x-2}$
4.	$y = 4 - x^2$	5.	$y = x^2 + 2x + 1$	6.	$y = -\sqrt{x} + 3$
7.	$y = -x^2 + 2x - 1$	8.	$y = \lvert x+2 \rvert$	9.	$y = 2\sqrt[3]{x} - 1$

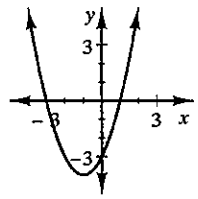
Answers

1.



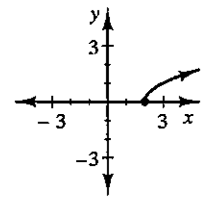
Parabola; intercepts $(0, 0)$, $(2,0)$; decreasing until $x = 1$ then increasing; minimum value at $(1, -1)$; $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to -1.

2.



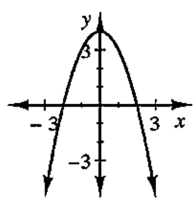
Parabola; intercepts $(-3, 0)$, $(1,0)$ and $(0, -3)$; decreasing until $x = -1$, then increasing; minimum value at $(-1, -4)$; $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to -4.

3.



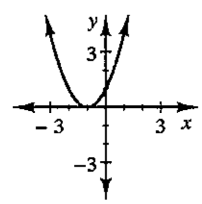
Half-parabola; starting point, intercept and minimum point $(2, 0)$; increasing for $x > 2$. Inputs can be any number greater than or equal to 2. Outputs are greater than or equal to 0.

4.



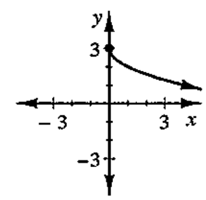
Parabola; intercepts $(-2, 0)$, $(2, 0)$ and $(0, 4)$; increasing for $x < 0$, decreasing for $x > 0$; maximum value at $(0, 4)$; $x = 0$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 4.

5.



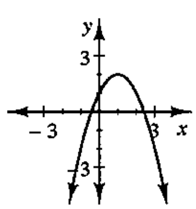
Parabola; intercept $(-1, 0)$; decreasing for $x < -1$, increasing for $x > -1$; minimum value at $(-1, 0)$; $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.

6.



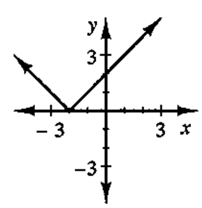
Half-parabola; starting point, intercept and maximum point $(0, 3)$; decreasing for $x > 0$. Inputs can be any number greater than or equal to 0. Outputs are less than or equal to 3.

7.



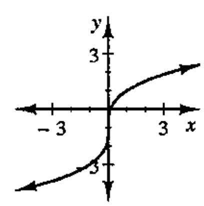
Parabola; intercepts $(-0.4, 0)$, $(2.4, 0)$ and $(0, 1)$; increasing for $x < 1$, decreasing for $x > 1$; maximum value at $(1, 2)$; $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 2.

8.



V-shape; intercepts $(-2, 0)$ and $(0, 2)$; decreasing for $x < -2$, increasing for $x > -2$; minimum value at $(-2, 0)$; $x = -2$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.

9.



S-shape; intercept $(0, 0)$; increasing for all x from left to right. Inputs and outputs can be any real number. There is no line of symmetry.

FUNCTIONS 1.2.4 through 1.2.5

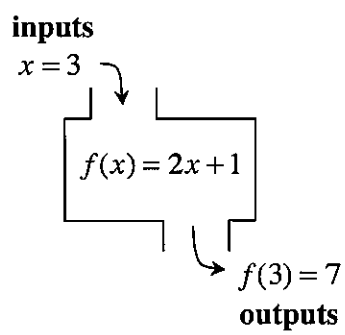
A relationship between the input values (usually x) and the output values (usually y) is called a function if for each input value, there is no more than one output value. Functions can be represented with an illustration of an input–output "machine," as shown in Lesson 1.2.3 of the textbook and in the diagram in Example 1 below.

The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

For additional information about functions, function notation and domain and range, see the Math Notes box in Lesson 1.2.5.

Example 1

Numbers, represented by a letter or symbol such as x, are input into the function machine labeled f one at a time, and then the function performs the operation on each input to determine each output, $f(x)$. For example, when $x = 3$ is put into the function f at right, the machine multiplies 3 by 2 and adds 1 to get the output, $f(x)$ which is 7. The notation $f(3) = 7$ shows that the function named f connects the input (3) with the output 7. This also means the point $(3, 7)$ lies on the graph of the function.



Example 2

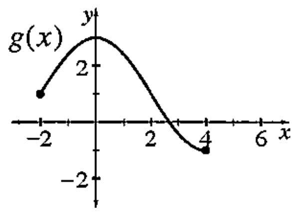
a. If $f(x) = \sqrt{x-2}$ then $f(11) = ?$ $\quad f(11) = \sqrt{11-2} = \sqrt{9} = 3$

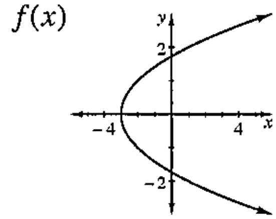
b. If $g(x) = 3 - x^2$ then $g(5) = ?$ $\quad g(5) = 3 - (5)^2 = 3 - 25 = -22$

c. If $f(x) = \dfrac{x+3}{2x-5}$ then $f(2) = ?$ $\quad f(2) = \dfrac{2+3}{2 \cdot 2 - 5} = \dfrac{5}{-1} = -5$

Example 3

A relation in which each input has only one output is called a **function**.





$g(x)$ is a function: each input (x) has only one output (y).
$g(-2)=1$, $g(0)=3$, $g(4)=-1$, and so on.

$f(x)$ is not a function: each input greater than -3 has two y-values associated with it.
$f(1)=2$ and $f(1)=-2$.

Example 4

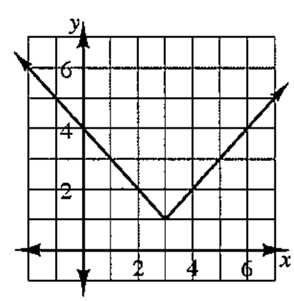
The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

In Example 3 above, the domain of $g(x)$ in Example 3 is $-2 \le x \le 4$, or "all numbers between -2 and 4." The range is $-1 \le x \le 3$ or "all numbers between -1 and 3."

The domain of $f(x)$ in Example 3 above is $x \ge -3$ or "any real number greater than or equal to 3," since the graph starts at -3 and continues forever to the right. Since the graph of $f(x)$ extends in both the positive and negative y directions forever, the range is "all real numbers."

Example 5

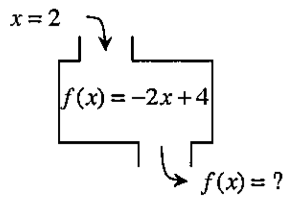
For the graph at right, since the x-values extend forever in both directions the domain is "all real numbers." The y-values start at 1 and go higher so the range is $y \ge 1$ or "all numbers greater or equal to 1."



Problems

Determine the outputs for the following relations and the given inputs.

1.



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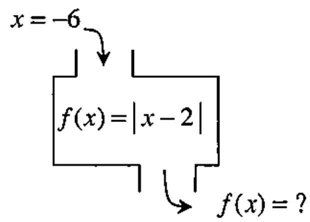

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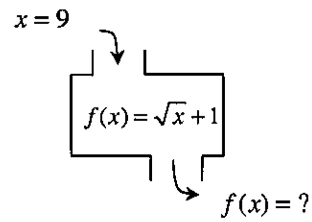


4. $f(x) = (5 - x)^2$
   $f(8) = ?$

5. $g(x) = x^2 - 5$
   $g(-3) = ?$

6. $f(x) = \frac{2x+7}{x^2-9}$
   $f(3) = ?$

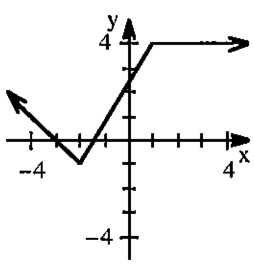
7. $h(x) = 5 - \sqrt{x}$
   $h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
   $h(9) = ?$

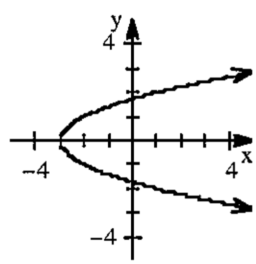
9. $f(x) = -x^2$
   $f(4) = ?$

Determine if each relation is a function. Then state its domain and range.

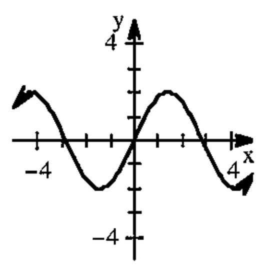
10.



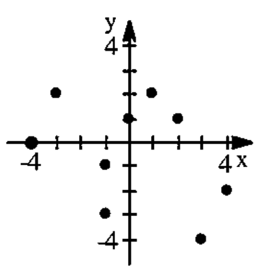
11.



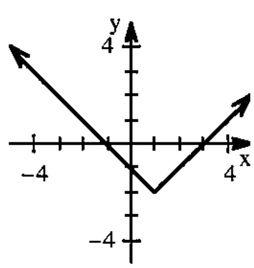
12.



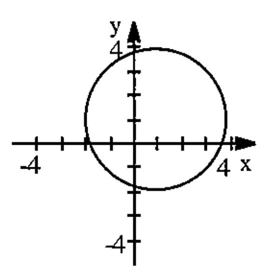
13.



14.



15.



Answers

1. 0
2. 8
3. 4
4. 9
5. 4
6. not possible
7. 2
8. not possible
9. -16
10. yes, each input has one output; domain is all numbers, range is $-1 \leq y \leq 4$
11. no, for example $x=0$ has two outputs; domain is $x \geq -3$, range is all numbers
12. yes; domain all numbers, range is $-2 \leq y \leq 2$
13. no; -1 has two outputs; domain is -4, $-3, -1, 0, 1, 2, 3, 4$, range is $-4, -3, -2, -1, 0, 1, 2$
14. yes; domain is all numbers, range is $y \geq -2$
15. no, many inputs have two outputs; domain is $-2 \leq x \leq 4$ range is $-2 \leq y \leq 4$

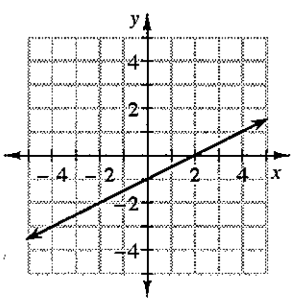
SLOPE—A MEASURE OF STEEPNESS 2.1.2 through 2.1.4

> Students used the equation $y = mx + b$ to graph lines and describe patterns in previous courses. Lesson 2.1.1 is a review. When the equation of a line is written in $y = mx + b$ form, the coefficient m represents the slope of the line. Slope indicates the direction of the line and its steepness. The constant b is the y-intercept, written $(0, b)$, and indicates where the line crosses the y-axis.
>
> For additional information about slope, see the Math Notes box in Lesson 2.1.4.

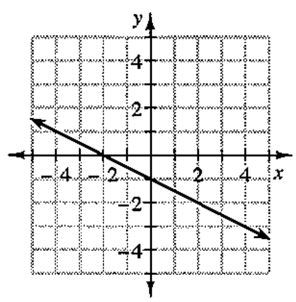
Example 1

If m is positive, the line goes upward from left to right. If m is negative, the line goes downward from left to right. If $m = 0$ then the line is horizontal. The value of b indicates the y-intercept.

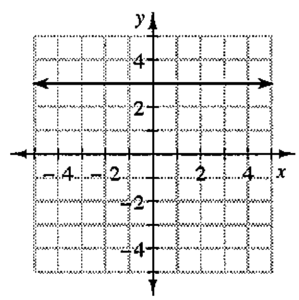
$y = \frac{1}{2}x - 1$



$y = -\frac{1}{2}x - 1$



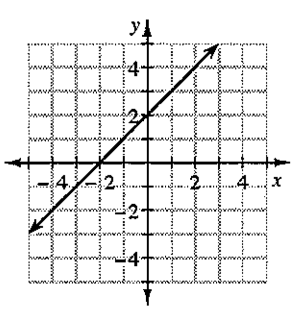
$y = 0x + 3$ or $y = 3$



Example 2

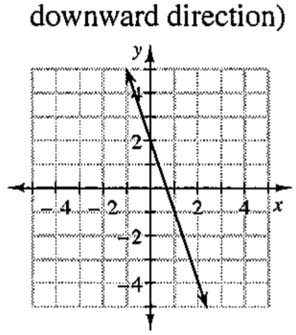
When $m = 1$, as in $y = x$, the line goes upward by one unit each time it goes over one unit to the right. Steeper lines have a larger m-value, that is, $m > 1$ or $m < -1$. Flatter lines have an m-value that is between 0 and 1, often in the form of a fraction. All three examples below have $b = 2$.

$y = x + 2$



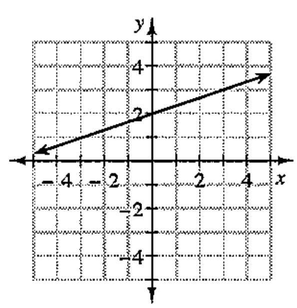
$y = -3x + 2$

(steeper and in the downward direction)



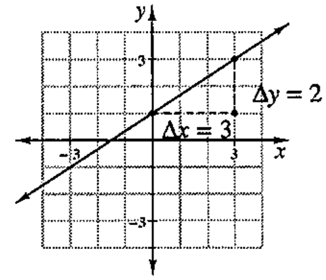
$y = \frac{1}{3}x + 2$

(less steep)



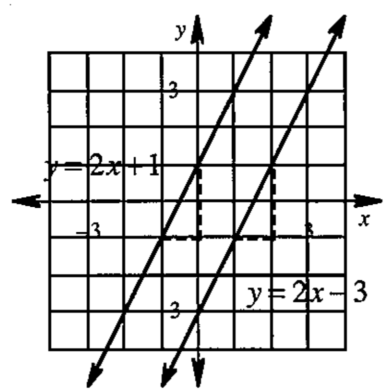
Example 3

Slope is written as a ratio. If the line is drawn on a set of axes, a *slope triangle* can be drawn between any two convenient points (usually where grid lines cross), as shown in the graph at right. Count the vertical distance (notated Δy) and the horizontal distance (notated Δx) on the dashed sides of the slope triangle. Write the distances in a ratio: slope $= m = \frac{\Delta y}{\Delta x} = \frac{2}{3}$. The symbol Δ means change. The order in the fraction is important: the numerator (top of the fraction) must be the vertical distance and the denominator (bottom of the fraction) must be the horizontal distance. The slope of a line is constant, so the slope ratio is the same for any two points on the line.



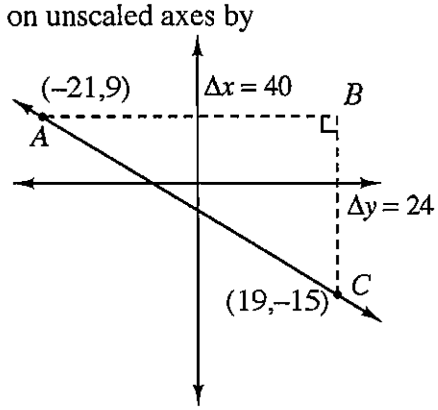
Parallel lines have the same steepness and direction, so they have the same slope, as shown in the graph at right.

If $\Delta y = 0$, then the line is horizontal and has a slope of zero, that is, $m = 0$. If $\Delta x = 0$, then the line is vertical and its slope is undefined, so we say that it has no slope.



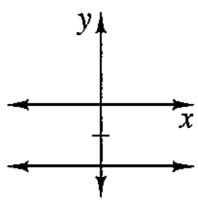
Example 4

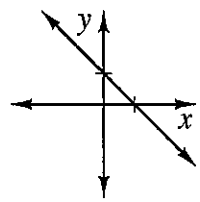
When the vertical and horizontal distances are not easy to determine, you can find the slope by drawing a generic slope triangle and using it to find the lengths of the vertical (Δy) and horizontal (Δx) segments. The figure at right shows how to find the slope of the line that passes through the points (–21, 9) and (19, –15). First graph the points on unscaled axes by approximating where they are located, then draw a slope triangle. Next find the distance along the vertical side by noting that it is 9 units from point B to the x-axis then 15 units from the x-axis to point C, so Δy is 24. Then find the distance from point A to the y-axis (21) and the distance from the y-axis to point B (19). Δx is 40. This slope is negative because the line goes downward from left to right, so the slope is $m = \frac{\Delta y}{\Delta x} = -\frac{24}{40} = -\frac{3}{5}$.

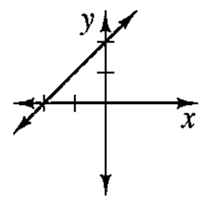


Problems

Is the slope of each line negative, positive, or zero?

1. 

2. 

3. 

Identify the slope in each equation. State whether the graph of the line is steeper or flatter than $y=x$ or $y=-x$, whether it goes up or down from left to right, or if it is horizontal or vertical.

4. $y=3x+2$
5. $y=-\frac{1}{2}x+4$
6. $y=\frac{1}{3}x-4$
7. $4x-3=y$
8. $y=-2+\frac{1}{2}x$
9. $3+2y=8x$
10. $y=2$
11. $x=5$
12. $6x+3y=8$

Without graphing, find the slope of each line based on the given information.

13. $\Delta y=27$ $\Delta x=-8$
14. $\Delta x=15$ $\Delta y=3$
15. $\Delta y=7$ $\Delta x=0$
16. Horizontal $\Delta=6$ Vertical $\Delta=0$
17. Between (5, 28) and (64, 12)
18. Between (–3, 2) and (5, -7)

Answers

1. zero
2. negative
3. positive
4. Slope = 3, steeper, up
5. Slope = $-\frac{1}{2}$, flatter, down
6. Slope = $\frac{1}{3}$, flatter, up
7. Slope = 4, steeper, up
8. Slope = $\frac{1}{2}$, flatter, up
9. Slope = 4, steeper, up
10. horizontal
11. vertical
12. Slope = –2, steeper, down
13. $-\frac{27}{8}$
14. $\frac{3}{15}=\frac{1}{5}$
15. undefined
16. 0
17. $-\frac{16}{59}$
18. $-\frac{9}{8}$

WRITING AN EQUATION GIVEN THE SLOPE AND A POINT ON THE LINE 2.3.1

In earlier work students used substitution in equations like $y = 2x + 3$ to find x and y pairs that make the equation true. Students recorded those pairs in a table, and then used them as coordinates to graph a line. Every point (x, y) on the line makes the equation true.

Later, students used the patterns they saw in the tables and graphs to recognize and write equations in the form of $y = mx + b$. The "b" represents the y-intercept of the line, the "m" represents the slope, while x and y represent the coordinates of any point on the line. Each line has a unique value for m and a unique value for b, but there are infinite (x, y) values for each linear equation.

The slope of the line is the same between any two points on that line. We can use this information to write equations without creating tables or graphs.

For additional information, see the Math Notes boxes in Lessons 2.2.2 and 2.2.3.

Example 1

What is the equation of the line with a slope of 2 that passes through the point (10, 17)?

Write the general equation of a line. $y = mx + b$

Substitute the values we know: m, x, and y. $17 = 2(10) + b$

Solve for b.

$17 = 20 + b$

$-3 = b$

Write the complete equation using the values of $m = 2$ and $b = -3$. $y = 2x - 3$

Example 2

This algebraic method can help us write equations of parallel lines. Parallel lines never intersect or meet. They have the *same* slope, m, but *different* y-intercepts, b.

What is the equation of the line parallel to $y=3x-4$ that goes through the point (2, 8)?

Write the general equation of a line.	$y=mx+b$
Substitute the values we know: $m, x,$ and y. Since the lines are parallel, the slopes are equal.	$8=3(2)+b$
Solve for b.	$8=6+b$ $2=b$
Write the complete equation.	$y=3x+2$

Problems

Write the equation of the line with the given slope that passes through the given point.

1. slope = 5, 3, 13)
2. slope = $-\frac{5}{3}$, $(3,-1)$
3. slope = -4, $(-2,9)$
4. slope = $\frac{3}{2}$, (6, 8)
5. slope = 3, $(-7,-23)$
6. slope = 2, $(\frac{5}{2},-2)$

Write the equation of the line *parallel* to the given line that goes through the given point.

7. $y=\frac{3}{5}x+2$ (0, 0)
8. $y=4x-1$ $(-2,-6)$
9. $y=-2x+5$ $(-4,-2)$
10. $y=4x+5$ $(-6,-28)$
11. $y=\frac{1}{3}x-1$ (6, 9)
12. $y=3x+8$ $(0,\frac{1}{2})$

Answers

1. $y=5x-2$
2. $y=-\frac{5}{3}x+4$
3. $y=-4x+1$
4. $y=\frac{3}{2}x-1$
5. $y=3x-2$
6. $y=2x-7$
7. $y=\frac{3}{5}x$
8. $y=4x+2$
9. $y=-2x-10$
10. $y=4x-4$
11. $y=\frac{1}{3}x+7$
12. $y=3x+\frac{1}{2}$

WRITING THE EQUATION OF A LINE GIVEN TWO POINTS 2.3.2

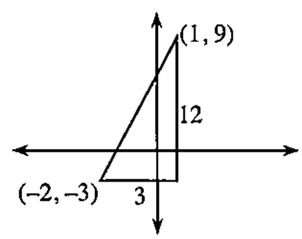
Students now have all the tools they need to find the equation of a line passing through two given points. Recall that the equation of a line requires a slope and a y-intercept in $y = mx + b$. Students can write the equation of a line from two points by creating a slope triangle and calculating $\frac{\Delta y}{\Delta x}$ as explained in Lessons 2.1.2 through 2.1.4.

For additional information, see the Math Notes box in Lesson 3.3.2. For additional examples and more practice, see the Checkpoint 5B materials.

Example 1

Write the equation of the line that passes through the points $(1, 9)$ and $(-2, -3)$.

Position the two points approximately where they belong on coordinate axes—you do not need to be precise. Draw a generic slope triangle.



Calculate slope $= \frac{\Delta y}{\Delta x} = \frac{12}{3} = 4$ using the given values of the two points.

$\frac{\Delta y}{\Delta x} = \frac{12}{3}$

$m = 4$

Write the general equation of a line. Substitute m and either one of the points into the equation. For example, use $(x, y) = (1, 9)$ and $m = 4$.

$y = mx + b$

$9 = 4(1) + b$

Solve for b.

$5 = b$

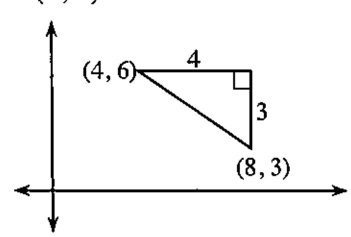
Write the complete equation.

$y = 4x + 5$

Example 2

Write the equation of the line that passes through the points $(8, 3)$ and $(4, 6)$.

Draw a generic slope triangle located approximately on coordinate axes. Approximate the locations of the given points.



Calculate $m = \frac{\Delta y}{\Delta x} = -\frac{3}{4}$. The slope is negative since the line goes down left to right.

$\frac{\Delta y}{\Delta x} = \frac{-3}{4}$

$m = -\frac{3}{4}$

Substitute m and either one of the points, for example $(8, 3)$, into the general equation for a line.

$y = mx + b$

$3 = -\frac{3}{4}(8) + b$

Solve for b.

$9 = b$

Write the complete equation.

$y = -\frac{3}{4}x + 9$

Problems

Write the equation of the line containing each pair of points.

1. $(1, 1)$ and $(0, 4)$
2. $(5, 4)$ and $(1, 1)$
3. $(1, 3)$ and $(-5, -15)$
4. $(-2, 3)$ and $(3, 5)$
5. $(2, -1)$ and $(3, -3)$
6. $(4, 5)$ and $(-2, -4)$
7. $(1, -4)$ and $(-2, 5)$
8. $(-3, -2)$ and $(5, -2)$
9. $(-4, 1)$ and $(5, -2)$

Answers

1. $y = -3x + 4$
2. $y = \frac{3}{4}x + \frac{1}{4}$
3. $y = 3x$
4. $y = \frac{2}{5}x + 3\frac{4}{5}$
5. $y = -2x + 3$
6. $y = \frac{3}{2}x - 1$
7. $y = -3x - 1$
8. $y = -2$
9. $y = -\frac{1}{3}x - \frac{1}{3}$

LAWS OF EXPONENTS 3.1.1 and 3.1.2

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic **laws of exponents** are listed here.

(1) $x^a \cdot x^b = x^{a+b}$ Examples: $x^3 \cdot x^4 = x^7$; $2^7 \cdot 2^4 = 2^{11}$

(2) $\frac{x^a}{x^b} = x^{a-b}$ Examples: $\frac{x^{10}}{x^4} = x^6$; $\frac{2^4}{2^7} = 2^{-3}$

(3) $\left(x^a\right)^b = x^{ab}$ Examples: $\left(x^4\right)^3 = x^{12}$; $\left(2x^3\right)^5 = 2^5 \cdot x^{15} = 32x^{15}$

(4) $x^0 = 1$ Examples: $2^0 = 1$; $(-3)^0 = 1$; $\left(\frac{1}{4}\right)^0 = 1$

(5) $x^{-n} = \frac{1}{x^n}$ Examples: $x^{-3} = \frac{1}{x^3}$; $y^{-4} = \frac{1}{y^4}$; $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(6) $\frac{1}{x^{-n}} = x^n$ Examples: $\frac{1}{x^{-5}} = x^5$; $\frac{1}{x^{-2}} = x^2$; $\frac{1}{3^{-2}} = 3^2 = 9$

(7) $x^{m/n} = \sqrt[n]{x^m}$ Examples: $x^{2/3} = \sqrt[3]{x^2}$; $y^{1/2} = \sqrt{y}$

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 3.1.2. For additional examples and practice, see the Checkpoint 5A problems in the back of the textbook.

Example 1

Simplify: $\left(2xy^3\right)\left(5x^2y^4\right)$

Reorder: $2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$

Using law (1): $10x^3y^7$

Example 2

Simplify: $\frac{14x^2y^{12}}{7x^5y^7}$

Separate: $\left(\frac{14}{7}\right) \cdot \left(\frac{x^2}{x^5}\right) \cdot \left(\frac{y^{12}}{y^7}\right)$

Using laws (2) and (5): $2x^{-3}y^5 = \frac{2y^5}{x^3}$

Example 3

Simplify: $\left(3x^2y^4\right)^3$

Using law (3): $3^3 \cdot \left(x^2\right)^3 \cdot \left(y^4\right)^3$

Using law (3) again: $27x^6y^{12}$

Example 4

Simplify: $\left(2x^3\right)^{-2}$

Using law (5): $\dfrac{1}{\left(2x^3\right)^2}$

Using law (3): $\dfrac{1}{2^2 \cdot \left(x^3\right)^2}$

Using law (3) again: $\dfrac{1}{4x^6}$

Example 5

Simplify: $\dfrac{10x^7y^3}{15x^{-2}y^3}$

Separate: $\left(\dfrac{10}{15}\right) \cdot \left(\dfrac{x^7}{x^{-2}}\right) \cdot \left(\dfrac{y^3}{y^3}\right)$

Using law (2): $\dfrac{2}{3}x^9y^0$

Using law (4): $\dfrac{2}{3}x^9 \cdot 1 = \dfrac{2}{3}x^9 = \dfrac{2x^9}{3}$

Problems

Simplify each expression. Final answers should contain no parenthesis, or negative exponents.

1. $y^5 \cdot y^7$
2. $b^4 \cdot b^3 \cdot b^2$
3. $8^6 \cdot 8^{-2}$
4. $(y^5)^2$
5. $(3a)^4$
6. $\frac{m^8}{m^3}$
7. $\frac{12m^8}{6m^{-3}}$
8. $(x^3y^2)^3$
9. $\frac{(y^4)^2}{(y^3)^2}$
10. $\frac{15x^2y^5}{3x^4y^5}$
11. $(4c^4)(ac^3)(3a^5c)$
12. $(7x^3y^5)^2$
13. $(4xy^2)(2y)^3$
14. $\left(\frac{4}{x^2}\right)^3$
15. $\frac{(2a^7)(3a^2)}{6a^3}$
16. $\left(\frac{5m^3n}{m^5}\right)^3$
17. $(3a^2x^3)^2(2ax^4)^3$
18. $\left(\frac{x^3y}{y^4}\right)^4$
19. $\left(\frac{6x^8y^2}{12x^3y^7}\right)^2$
20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$
21. x^{-3}
22. $2x^{-3}$
23. $(2x)^{-3}$
24. $(2x^3)^0$
25. $5^{1/2}$
26. $\left(\frac{2x}{3}\right)^{-2}$

Answers

1.	y^{12}	2.	b^9	3.	8^4
4.	y^{10}	5.	$81a^4$	6.	m^5
7.	$2m^{11}$	8.	x^9y^6	9.	y^2
10.	$\frac{5}{x^2}$	11.	$12a^6c^8$	12.	$49x^6y^{10}$
13.	$32xy^5$	14.	$\frac{64}{x^6}$	15.	a^6
16.	$\frac{125n^3}{m^6}$	17.	$72a^7x^{18}$	18.	$\frac{x^{12}}{y^{12}}$
19.	$\frac{x^{10}}{4y^{10}}$	20.	$16x^{10}y^5$	21.	$\frac{1}{x^3}$
22.	$\frac{2}{x^3}$	23.	$\frac{1}{8x^3}$	24.	1
25.	$\sqrt{5}$	26.	$\frac{9}{4x^2}$		

EQUATIONS ↔ ALGEBRA TILES 3.2.1

An equation mat can be used together with algebra ties to represent the process of solving an equation. For assistance with Lesson 3.2.1, see Lessons A.1.1 through A.1.9 in this Parent Guide.

See the Math Notes box in Lesson A.1.8 (in the Appendix chapter of the textbook) and in Lesson 3.2.1 for a list of all the "legal" moves and their corresponding algebraic equivalents. Also see the Math Notes box in Lesson A.1.9 (in the Appendix chapter of the textbook) for checking a solution.

For additional examples and practice, see the Checkpoint 1 materials at the back of the textbook.

Two ways to find the area of a rectangle are: as a product of the (height) · (base) or as the sum of the areas of individual pieces of the rectangle. For a given rectangle these two areas must be the same, so **Area as a product = Area as a sum**. Algebra tiles, and later, generic rectangles, provide area models to help multiply expressions in a visual, concrete manner.

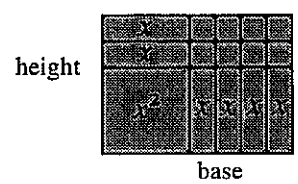
For additional information, see the Math Notes boxes in Lessons 3.2.2, 3.2.3, and 3.3.3. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

Example 1: Using Algebra Tiles

The algebra tile pieces x^2+6x+8 are arranged into a rectangle as shown at right. The area of the rectangle can be written as the **product** of its base and height or as the **sum** of its parts.

$$\underbrace{(x+4)}_{base}\underbrace{(x+2)}_{height} = \underbrace{x^2+6x+8}_{area}$$

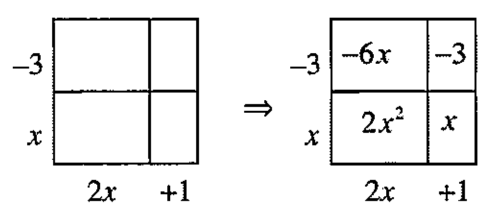
area as a **product** area as a **sum**



Example 2: Using Generic Rectangles

A generic rectangle allows us to organize the problem in the same way as the first example without needing to draw the individual tiles. It does not have to be drawn accurately or to scale.

Multiply $\underbrace{(x-3)}_{base}\underbrace{(2x+1)}_{height}$.



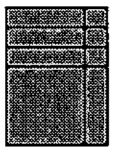
$\Rightarrow (x-3)(2x+1) = 2x^2-5x-3$

area as a product area as a sum

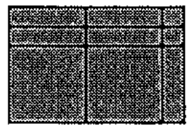
Problems

Write a statement showing: **area as a product** equals **area as a sum.**

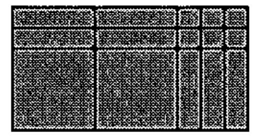
1.



2.



3.



4.

x	x^2	$3x$
-5	$-5x$	-15
	x	$+3$

5.

6	$18y$	$-12x$
	$3y$	$-2x$

6.

$3y$	$3xy$	$12y$
-2	$-2x$	-8
	x	$+4$

Multiply.

7. $(3x+2)(2x+7)$
8. $(2x-1)(3x+1)$
9. $(2x)(x-1)$
10. $(2y-1)(4y+7)$
11. $(y-4)(y+4)$
12. $(y)(x-1)$
13. $(3x-1)(x+2)$
14. $(2y-5)(y+4)$
15. $(3y)(x-y)$
16. $(3x-5)(3x+5)$
17. $(4x+1)^2$
18. $(x+y)(x+2)$
19. $(2y-3)^2$
20. $(x-1)(x+y+1)$
21. $(x+2)(x+y-2)$

Answers

1. $(x+1)(x+3)=x^2+4x+3$
2. $(x+2)(2x+1)=2x^2+5x+2$
3. $(x+2)(2x+3)=2x^2+7x+6$
4. $(x-5)(x+3)=x^2-2x-15$
5. $6(3y-2x)=18y-12x$
6. $(x+4)(3y-2)=3xy-2x+12y-8$
7. $6x^2+25x+14$
8. $6x^2-x-1$
9. $2x^2-2x$
10. $8y^2+10y-7$
11. y^2-16
12. $xy-y$
13. $3x^2+5x-2$
14. $2y^2+3y-20$
15. $3xy-3y^2$
16. $9x^2-25$
17. $16x^2+8x+1$
18. $x^2+2x+xy+2y$
19. $4y^2-12y+9$
20. $x^2+xy-y-1$
21. $x^2+xy+2y-4$

SOLVING EQUATIONS WITH MULTIPLICATION OR ABSOLUTE VALUE 3.3.1

To **solve an equation with multiplication**, first use the Distributive Property or a generic rectangle to rewrite the equation without parentheses, then solve in the usual way. For additional information, see the Math Notes box in Lesson 3.3.1. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

To **solve an equation with absolute value**, first break the problem into two cases since the quantity inside the absolute value can be positive or negative. Then solve each part in the usual way.

Example 1

Solve $6(x+2)=3(5x+1)$

Use the Distributive Property. $6x+12=15x+3$

Subtract $6x$. $12=9x+3$

Subtract 3. $9=9x$

Divide by 9. $1=x$

Example 2

Solve $x(2x-4)=(2x+1)(x+5)$

Rewrite the equation using the Distributive Property on left side of equal sign and generic rectangles on right of equal sign.

$2x^2-4x=$

5	$10x$	5
x	$2x^2$	x
	$2x$	$+1$

$2x^2-4x=2x^2+11x+5$

Subtract $2x^2$ from both sides. $-4x=11x+5$

Subtract $11x$ from both sides. $-15x=5$

Divide by -15. $x=\frac{5}{-15}=-\frac{1}{3}$

Example 3

Solve $|2x-3|=7$

Separate into two cases.	$2x-3=7$	or	$2x-3=-7$
Add 3.	$2x=10$	or	$2x=-4$
Divide by 2.	$x=5$	or	$x=-2$

Problems

Solve each equation.

1. $3(c+4)=5c+14$
2. $x-4=5(x+2)$
3. $7(x+7)=49-x$
4. $8(x-2)=2(2-x)$
5. $5x-4(x-3)=8$
6. $4y-2(6-y)=6$
7. $2x+2(2x-4)=244$
8. $x(2x-4)=(2x+1)(x-2)$
9. $(x-1)(x+7)=(x+1)(x-3)$
10. $(x+3)(x+4)=(x+1)(x+2)$
11. $2x-5(x+4)=-2(x+3)$
12. $(x+2)(x+3)=x^2+5x+6$
13. $(x-3)(x+5)=x^2-7x-15$
14. $(x+2)(x-2)=(x+3)(x-3)$
15. $\frac{1}{2}x(x+2)=\left(\frac{1}{2}x+2\right)(x-3)$
16. $|3x+2|=11$
17. $|5-x|=9$
18. $|3-2x|=7$
19. $|2x+3|=-7$
20. $|4x+1|=10$

Answers

1. $c=-1$
2. $x=-3.5$
3. $x=0$
4. $x=2$
5. $x=-4$
6. $y=3$
7. $x=42$
8. $x=2$
9. $x=0.5$
10. $x=-2.5$
11. $x=-14$
12. all numbers
13. $x=0$
14. no solution
15. $x=-12$
16. $x=3$ or $-\frac{13}{3}$
17. $x=-4$ or 14
18. $x=-2$ or 5
19. no solution
20. $x=\frac{9}{4}$ or $-\frac{11}{4}$

REWRITING MULTI-VARIABLE EQUATIONS 3.3.2

Rewriting equations with more than one variable uses the same "legal" moves process as solving an equation with one variable in Lessons 3.2.1, A.1.8, and A.1.9. The end result is often not a number, but rather an algebraic expression containing numbers and variables.

For "legal" moves, see the Math Notes box in Lesson 3.2.1. For additional examples and more practice, see the Checkpoint 6A materials at the back of the textbook.

Example 1

Solve for y	$3x-2y=6$
Subtract $3x$	$-2y=-3x+6$
Divide by -2	$y=\frac{-3x+6}{-2}$
Simplify	$y=\frac{3}{2}x-3$

Example 2

Solve for y	$7+2(x+y)=11$
Subtract 7	$2(x+y)=4$
Distribute the 2	$2x+2y=4$
Subtract $2x$	$2y=-2x+4$
Divide by 2	$y=\frac{-2x+4}{2}$
Simplify	$y=-x+2$

Example 3

Solve for x	$y=3x-4$
Add 4	$y+4=3x$
Divide by 3	$\frac{y+4}{3}=x$

Example 4

Solve for t	$I=prt$
Divide by pr	$\frac{I}{pr}=t$

Problems

Solve each equation for the specified variable.

1. Solve for y: $5x+3y=15$
2. Solve for x: $5x+3y=15$
3. Solve for w: $2l+2w=P$
4. Solve for m: $4n=3m-1$
5. Solve for a: $2a+b=c$
6. Solve for a: $b-2a=c$
7. Solve for p: $6-2(q-3p)=4p$
8. Solve for x: $y=\frac{1}{4}x+1$
9. Solve for r: $4(r-3s)=r-5s$

Answers (Other equivalent forms are possible.)

1.	$y=-\frac{5}{3}x+5$	2.	$x=-\frac{3}{5}y+3$	3.	$w=-l+\frac{P}{2}$
4.	$m=\frac{4n+1}{3}$	5.	$a=\frac{c-b}{2}$	6.	$a=\frac{c-b}{-2}$ *or* $\frac{b-c}{2}$
7.	$p=q-3$	8.	$x=4y-4$	9.	$r=\frac{7s}{3}$