

## Finding the Equation of a Line Given Two Points

Most people when asked, “What is the equation of a line?”, will answer, “ $y = mx + b$ ”. This is the equation of a line in what is called slope-intercept form where “ $m$ ” is the slope and “ $b$ ” is the y-intercept. So, how do you find the equation of a line? There are several different ways that you can find the equation of a line. I find the equation of a line everytime by following the same three steps:

- Step 1: Find the slope of the line.  
Step 2: Use the slope to find the y-intercept.  
Step 3: Use steps 1 and 2 to write the answer.

I will explain these steps by looking at several examples. Please understand that there are often several different ways to complete each math problem, but I have found through the years that students are most successful when they do problems the same way each and every time they attempt the problem. If you know a different way to find the answer that is great, but I am going to show how to do the problem one way and use the same technique everytime I see this problem.

**Example 1:** Find the equation of the line passing through the points  $(-1, -2)$  and  $(2, 7)$ .

**Step 1:** Find the slope of the line.

To find the slope of the line passing through these two points we need to use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{2 - (-1)} = \frac{9}{3} = 3$$

So the slope of the line passing through these two points is 3.

**Step 2:** Use the slope to find the y-intercept.

Now that we know the slope of the line is 3 we can plug the slope into the equation and we get:

$$y = 3x + b$$

Next choose one of the two points to plug in for the values of  $x$  and  $y$ . It does not matter which one of the two points you choose because you should get the same answer in either case. I generally just choose the first point listed so I don't have to worry about which one I should choose.

$$(-1, -2) \rightarrow -2 = 3(-1) + b \quad \text{Multiply to simplify the problem.}$$

$$-2 = -3 + b \quad \text{Solve for } b \text{ and you will have the y-intercept.}$$

$$b = 1$$

**Step 3:** Write the answer.

Using the slope of 3 and the y-intercept of 1, the answer is:

$$y = 3x + 1$$

**Example 2:** Find the equation of the line passing through the points  $(-2, 5)$  and  $(4, -3)$ .

**Step 1:** Find the slope of the line.

To find the slope of the line passing through these two points we need to use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{4 - (-2)} = -\frac{8}{6} = -\frac{4}{3}$$

So the slope of the line passing through these two points is  $-4/3$ .

**Step 2:** Use the slope to find the y-intercept.

Now that we know the slope of the line is  $-4/3$  we can plug the slope into the equation and we get:

$$y = -\frac{4}{3}x + b$$

Next choose one of the two points to plug in for the values of x and y. It does not matter which one of the two points you choose, you should get the same answer in either case.

$$(-2, 5) \rightarrow 5 = \left(-\frac{4}{3}\right)(-2) + b$$

Multiply to simplify the problem.

$$5 = \frac{8}{3} + b \rightarrow 3\left(5 = \frac{8}{3} + b\right)$$

At this point if you are not good with fractions, multiply the entire problem by 3 (the common denominator) and the fractions will go away.

$$15 = 8 + 3b$$

Solve for b and you will have the y-intercept.

$$b = \frac{7}{3}$$

**Step 3:** Write the answer.

Using the slope of  $-4/3$  and the y-intercept of  $7/3$ , the answer is:

$$y = -\frac{4}{3}x + \frac{7}{3}$$

**Example 3:** Find the equation of the line passing through the points  $(-5, -2)$  and  $(1, 5)$ .

**Step 1:** Find the slope of the line.

To find the slope of the line passing through these two points we need to use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{1 - (-5)} = \frac{7}{6}$$

So the slope of the line passing through these two points is  $7/6$ .

**Step 2:** Use the slope to find the y-intercept.

Now that we know the slope of the line is  $7/6$  we can plug the slope into the equation and we get:

$$y = \frac{7}{6}x + b$$

Next choose one of the two points to plug in for the values of x and y. It does not matter which one of the two points you choose, you should get the same answer in either case.

$$(-5, -2) \rightarrow -2 = \left(\frac{7}{6}\right)(-5) + b$$

Multiply to simplify the problem.

$$-2 = -\frac{35}{6} + b \rightarrow 6\left(-2 = -\frac{35}{6} + b\right)$$

Multiply the entire problem by 6 (the common denominator) and the fractions will go away.

$$-12 = -35 + 6b$$

Solve for b and you will have the y-intercept.

$$b = \frac{23}{6}$$

**Step 3:** Write the answer.

Using the slope of  $7/6$  and the y-intercept of  $23/6$ , the answer is:

$$y = \frac{7}{6}x + \frac{23}{6}$$

**Example 4:** Find the equation of the line passing through the points  $(-4, 7)$  and  $(3, 2)$ .

**Step 1:** Find the slope of the line.

To find the slope of the line passing through these two points we need to use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 7}{3 - (-4)} = -\frac{5}{7}$$

So the slope of the line passing through these two points is  $-5/7$ .

**Step 2:** Use the slope to find the y-intercept.

Now that we know the slope of the line is  $-5/7$  we can plug the slope into the equation and we get:

$$y = -\frac{5}{7}x + b$$

Next choose one of the two points to plug in for the values of x and y. It does not matter which one of the two points you choose, you should get the same answer in either case.

$$(-4, 7) \rightarrow 7 = \left(-\frac{5}{7}\right)(-4) + b$$

Multiply to simplify the problem.

$$7 = \frac{20}{7} + b \rightarrow 7 \left(7 = \frac{20}{7} + b\right)$$

Multiply the entire problem by 7 (the common denominator) and the fractions will go away.

$$49 = 20 + 7b$$

Solve for b and you will have the y-intercept.

$$b = \frac{29}{7}$$

**Step 3:** Write the answer.

Using the slope of  $-5/7$  and the y-intercept of  $29/7$ , the answer is:

$$y = -\frac{5}{7}x + \frac{29}{7}$$