Winter Break Packet Directions and Credit: **Name:** ________________________________ **#:**_______

Here are the directions for your MAGICAL Winter Break Packet! MAGICAL because if you understand the MAGIC of exponents, you'll be able to factor with ease and understand the power of numbers!

In Unit 3, we are going to be working with exponents. It is imperative that you have a solid working foundation with exponents to be successful in the chapter and beyond. I am using you Winter Break packet as a way to flip the classroom and have you complete a few lessons on exponents and operations prior to beginning the unit. There are 4 lessons.

1.1: Operations with Exponents

1.2: Negative Exponents

1.3: Negative Exponents Operations

5.5: Solving Exponents Equations

IN ORDER TO RECEIVE CREDIT, you must complete all of the following, in the following order. Complete one lesson at a time, including the notes and video component for each lesson before moving onto the next.

1) Lessons 1.1., 1.2, 1.3, and 5.5

2) Read, highlight, annotate the notes section of that lesson.

3) Create a sheet for Cornell Notes for EACH LESSON. After completely reading through the notes, rewrite the notes into your Cornell Notes. DO NOT COPY EVERYTHING! Put the important concepts into your own words. Write down examples. Write any questions you might have. THESE MUST BE TURNED IN WITH YOUR PACKET IF YOU WANT CREDIT!

4)Watch the video(s) that correspond with that lesson posted on our website. Write more notes into your Cornell Notes. When a problem is given in the video, pause the video, write down the problem (if you need extra paper, you can attach more), try it, then play the video to see if you got it right.

5) Complete the lesson. Show necessary work! Box your answers!

6) After doing the problems, review your notes, review the lesson and write a summary on your Cornell Notes for that lesson!

7) Once you have completed all of the lessons, go back, review your work and the lessons and create a front and back study guide sheet that contains all of the important topics from these lessons. You should include all the exponents laws (if, after doing all the lessons in the way I laid out above, you are still unsure about them, Google it! Watch some more videos, visit a few websites!) There will be a quiz when we return to see if 1) you did the work and 2) you learned what you needed to from the lessons.

There is no specialized credit structure because everyone needs to do all of the work. So work hard and do it right!

If you can, it is best to spread the work out. You can also do some Khan Academy over the break for those of you with parents eager to have your brains stay sharp.

Good luck! Ms. Cori

\- You must watch accompanying videos! Posted on website. Read all notes!

1.1 Operations with Exponents

Must be correct and complete for credit.

First let's start with a review of what exponents are. Recall that 3^4 means taking four 3's and multiplying them together. So we know that $3^4 = 3 \times 3 \times 3 \times 3 = 81$. You might also recall that in the number 3^4, three is called the base and four is called the exponent. Other reminds include that any number to the zero power is equal to one (so $5^0 = 1$) and any number is equal to itself to the first power (so $5^1 = 5$).

Sometimes it is easier to leave a number written as an exponent. For example, it is much easier to write 5^{20} instead of 95,367,431,640,625. Not only is sometimes simpler to write a number using exponents, but many operations are easier when the numbers are written as exponents.

Multiplying Numbers with the Same Base

Let's examine the problem $3^4 \times 3^4$ and write the answer as an exponent. Yes, we could multiply it out as a standard form number, $81 \times 81 = 6561$, but let's keep it in exponential form to see if it is any easier.

First, let's expand the problem: $3^4 \times 3^4 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$. Notice that the only operation that is happening here is multiplication and that we are multiplying the same number. That means we can say the following: $3^4 \times 3^4 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^8$. In short we see that $3^4 \times 3^4 = 3^8$. Do you see a rule that we could generalize from this?

Let's look at another example but this time with a variable.

$$y^7 \times y^4 = (y \times y \times y \times y \times y \times y \times y) \times (y \times y \times y \times y) = y^{11}$$

Can you find a rule that we can use when multiplying two exponent numbers with the same base? Yes, we can add the exponents. In other words, $z^5 \times z^6 = z^{5+6} = z^{11}$ would be a quicker way to show work for this problem. Generalizing this, we have the rule that $x^a \times x^b = x^{a+b}$.

Will this work with numbers without the same base? Let's find out by looking at $5^2 \times 2^3$. Many people think that $5^2 \times 2^3 = 10^5$, but we know that $5^2 \times 2^3 = 25 \times 8 = 200$ and that $10^5 = 100{,}000$. So we see that $5^2 \times 2^3 = 10^5$ is not true. Therefore we know that we can only add the exponents when we have the same base.

In fact, if asked to simplify $4^2 \times 7^2$ we would either have to multiply it out as a regular number or else leave it alone if we wanted it written using exponents.

Dividing Numbers with the Same Base

If multiplying numbers with the same base meant that we could add the exponents, what rule do you think we will discover when dividing numbers with the same exponent? Let's find out by looking at an example.

$$\frac{4^7}{4^5} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4}$$

Note that since only multiplication and division is happening, five of fours in the denominator will "cancel" (they actually become one since four divided by four is one, we just call it "canceling") with five of the fours in the numerator. That means we get the following:

$$\frac{4^7}{4^5} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4} = \frac{4 \times 4}{1} = 4^2$$

Let's look at one more example using variables before generalizing a rule for dividing exponent numbers with the same base.

$$\frac{q^6}{q^2} = \frac{q \times q \times q \times q \times q \times q}{q \times q} = \frac{q \times q \times q \times q}{1} = q^4$$

It looks like our rule is similar to the multiplication of exponent numbers with the same base, but this time we subtract the exponents. This gives us the general rule of $\frac{x^a}{x^b} = x^{a-b}$. For now we will only deal with division cases where the numerator exponent is larger than the denominator, but think ahead to what would happen if the denominator's exponent were larger. What do you think would happen?

A Power to a Power

We can also take exponents themselves to a power. For example, think of the problem $(2^3)^2$. Following our order of operations, we know that we have to do the parentheses first which means we get $(2^3)^2 = 8^2 = 64$. However, what if we wanted to leave our answer as a number to a power? Note the following:

$$(2^3)^2 = (2^3)(2^3) = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$$

Again, can you see a rule here? Let's look at an example with a variable to help again.

$$(g^4)^3 = (g^4)(g^4)(g^4) = (g \times g \times g \times g) \times (g \times g \times g \times g) \times (g \times g \times g \times g) = g^{12}$$

For a power to a power when using the same base we get the rule that you can multiply the exponents. This generalizes to $(x^a)^b = x^{ab}$.

Lesson 1.1

Perform the following operations leaving your answer as a number to a power. Remember that the parentheses can mean multiply as well.

1. $5^3 \times 5^7$
2. $(12^9)(12^0)$
3. $\frac{(t^5)(t^4)}{t^2}$
4. $\frac{4^{13}}{4^7} \times 4^{10}$
5. $\frac{f^5}{f}$
6. $\frac{u^{11}}{u^4}$
7. $(5^4)^5$
8. $(b^3)^6 \times (b^2)^9$
9. $(j^{11})^5$

Evaluate, meaning multiply out the exponents.

10. $3^2 \times 3^2$
11. $\frac{(2^{10})(2^2)}{2^9}$
12. $\frac{(5^3)^2}{5^4}$
13. $\frac{4^{12}}{4^{10}}$
14. $(5^3)^1 \times 5^0$
15. $(1^4)^2$

Determine if the following equations are true. Justify your answer.

16. $12^2 \times 12^7 = 12^6 \times 12^3$
17. $\frac{x^8}{x^3} = \frac{x^5}{x}$
18. $(t^5)^2 = (t^2)^5$
19. $(5^{10})^2 = (5^5)^5$
20. $\frac{6^0 \times 6^8}{6^4} = \frac{6^4}{6^0}$
21. $m^5 \times m^5 = (m^{10})^0$
22. $\frac{k^6}{k^2} = k^2 \times k^6$
23. $\frac{(7^4)^2}{7^3} = 7^3 \times 7^2$
24. $\frac{3 \times 3^4}{3^4} = (3^5)^1$

Determine the appropriate exponent to make the equation true.

25. $2^5 \times 2^{\square} = 2^3 \times 2^3$
26. $\frac{p^6}{p^2} = \frac{p^7}{p^{\square}}$
27. $(3^4)^3 = (3^6)^{\square}$
28. $(5^{10})^2 = (5^{\square})^5$
29. $\frac{b^2 \times b^8}{b^{\square}} = \frac{b^7}{b^3}$
30. $9^{\square} \times 9^8 = (9^3)^5$
31. $\frac{h^{\square}}{h^2} = h^3 \times h^5$
32. $\frac{(6^{11})^{\square}}{6^6} = 6^8 \times 6^8$
33. $\frac{3^{\square} \times 3^9}{3^2} = (3^7)^1$

You must watch accompanying videos! Posted on website! Read all notes!

1.2 Negative Exponents

Must be correct + complete for credit.

Last time we learned that when we divide exponent numbers with the same base we can subtract the exponents. We only examined problems where the numerator had a higher exponent than the denominator, but what would happen if the denominator had the higher exponent? Let's look.

$$\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5}$$

Notice that three of the fives will "cancel" (remember that they really become one because five divided by five is one). That means we are left with the following:

$$\frac{5^3}{5^5} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5 \times 5} = \frac{1}{5^2}$$

However, by following our rule from last time we know that we can also subtract the exponents which gives us:

$$\frac{5^3}{5^5} = 5^{-2}$$

Since $\frac{5^3}{5^5} = 5^{-2}$ and also $\frac{5^3}{5^5} = \frac{1}{5^2}$, by the transitive property we know that $5^{-2} = \frac{1}{5^2}$. We can now generalize this rule to say the following for any positive integer n:

$$x^{-n} = \frac{1}{x^n}$$

Negative Exponent as the Reciprocal

Another helpful way to think about negative exponents is as the reciprocal. Remember that the reciprocal of an integer is one over that integer because a number times its reciprocal must equal one. So 4^{-2} means the reciprocal of 4^2 which is $\frac{1}{4^2}$ or $\frac{1}{16}$. (Notice that $4^2 \times \frac{1}{4^2} = 1$ proving that we have the reciprocal.)

One last note is that except for scientific notation, we never leave negative exponents in a solution. We also take the reciprocal so that our exponent is positive. Let's look at a few more examples. Notice that we can evaluate the integer powers, but the variables to a power we have to leave the exponent.

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ $\quad$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\quad$ $10^{-5} = \frac{1}{10^5} = \frac{1}{100{,}000}$ $\quad$ $13^{-1} = \frac{1}{13^1} = \frac{1}{13}$

$q^{-3} = \frac{1}{q^3}$ $\quad$ $w^{-7} = \frac{1}{w^7}$ $\quad$ $g^{-11} = \frac{1}{g^{11}}$ $\quad$ $j^{-1} = \frac{1}{j^1} = \frac{1}{j}$

Lesson 1.2

Evaluate the following negative exponents giving your answer as a fraction.

1. 5^{-3} 2. 2^{-2} 3. 3^{-2} 4. 7^{-2} 5. 4^{-3} 6. 10^{-3}

7. 10^{-2} 8. 1^{-14} 9. 6^{-2} 10. 2^{-4} 11. 9^{-1} 12. 5^{-2}

13. 10^{-4} 14. 8^{-1} 15. 3^{-4} 16. 6^{-1} 17. 4^{-2} 18. 11^{-1}

Simplify the negative exponents giving your answer as a fraction.

19. a^{-3} 20. b^{-2} 21. c^{-5} 22. d^{-6} 23. f^{-11} 24. g^{-13}

25. h^{-1} 26. j^{-4} 27. k^{-20} 28. m^{-9} 29. n^{-7} 30. p^{-10}

You must watch accompanying videos on Website! Read all notes.
Must be correct and complete for credit!

1.3 Negative Exponents Operations

Now that we know negative exponents mean reciprocal, we can perform operations with negative exponents just like we did with positive exponents. Consider the following example of the multiplication rule. Notice that we still added the exponents, but just need to write our answer as a fraction if we have a negative exponent left after multiplication.

$$(5^3)(5^{-5}) = 5^{3+-5} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$(4^7)(4^{-5}) = 4^{7+-5} = 4^2 = 16$$

Now let's look at a division example. Remember that we found we can subtract the exponents as long as we have the same base.

$$\frac{5^2}{5^{-2}} = 5^{2--2} = 5^4 = 625$$

$$\frac{4^{-1}}{4^3} = 4^{-1-3} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$$

Finally we can see that the power to a power rule still works with negative exponents. We simply multiply the exponents.

$$(2^3)^{-2} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$(3^{-2})^{-2} = 3^4 = 81$$

Lesson 1.3

Evaluate the following exponents operations giving your answer as a fraction where necessary.

1. $5^3 \times 5^{-4}$
2. $(12^9)(12^{-7})$
3. $\frac{(t^{-5})(t^4)}{t^2}$
4. $\frac{4^3}{4^{-7}} \times 4^{-10}$
5. $\frac{f^5}{f^{-1}}$
6. $(y^{-4})^{-5}$
7. $(2^3)^{-6} \times (2^2)^7$
8. $12^2 \times 12^{-4}$
9. $\frac{(k^{-3})^2}{k^4}$
10. $\frac{4^{-2}}{4}$
11. $(5^{-3})^2 \times 5^9$
12. $(0^{-4})^{10}$

Determine if the following equations are true. Justify your answer.

13. $12^{-2} \times 12^7 = 12^{-8} \times 12^3$
14. $\frac{x^{-5}}{x^{-3}} = \frac{x^5}{x^7}$
15. $(t^{-5})^2 = (t^{-2})^5$
16. $(5^{10})^2 = (5^{-5})^{-4}$
17. $\frac{6^{-6} \times 6^8}{6^4} = \frac{6^{-2}}{6^0}$
18. $m^7 \times m^7 = (m^{-7})^2$
19. $\frac{k^{-6}}{k^2} = k^2 \times k^{-10}$
20. $\frac{(7^{-4})^2}{7^3} = 7 \times 7^{12}$
21. $\frac{3 \times 3^4}{3^{10}} = (3^5)^{-1}$

Determine the appropriate exponent to make the equation true.

22. $2^5 \times 2^{\square} = 2^{-6} \times 2^3$
23. $\frac{p^6}{p^{-2}} = \frac{p^{\square}}{p^2}$
24. $(3^{-4})^3 = (3^{-2})^{\square}$
25. $(5^{12})^{-2} = (5^3)^{\square}$
26. $\frac{b^{-2} \times b^8}{b^5} = \frac{b^{\square}}{b^3}$
27. $9^2 \times 9^{-8} = (9^{\square})^3$
28. $\frac{h^{-2}}{h^{\square}} = h^3 \times h^{-5}$
29. $\frac{(6^2)^{\square}}{6^6} = 6^{-8} \times 6^8$
30. $\frac{3^{-4}}{3^{\square} \times 3^9} = (3^7)^{-1}$

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5.5 Solving Exponent Equations

It may have become evident that squaring a number and taking the square roots are opposites. In fact, we call them inverse operations. In the same way, cubing a number and taking the cube root are inverse operations. We can use that fact to solve equations that involve exponents.

Solving $x^2 = p$ where p is rational

Let's start by looking at equations where p is not only rational, but also an integer. Remember that our goal in solving equations is to isolate the variable so that we will know what value the variable is equal to. So we are trying to "undo" everything that happens to the variable. Consider the following example. In order to get the variable by itself, we have to undo the square. We use the square root to do so and notice that whatever we do to one side of the equation, we must do to the other to maintain equality.

$$x^2 = 16$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

Notice that we had to give the answer of plus or minus four. Plug in both values for x to verify this is true. When $x = 4$ we see that $(4)^2 = 16$, and when $x = -4$ we see that $(-4)^2 = 16$.

It is also possible to solve an equation where the answer is a fraction. For example, consider the following problem:

$$x^2 = \frac{4}{9}$$

$$\sqrt{x^2} = \sqrt{\frac{4}{9}}$$

We know we need to take the square root of both sides of the equation, but how do we take the square root of a fraction? In particular, what fraction times itself is $\frac{4}{9}$? Note that $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ and therefore $\sqrt{\frac{4}{9}} = \frac{2}{3}$. This means that to take the square root of a fraction, we can take the square root of the numerator and the square root of the denominator. This is because when we multiply fractions the numerator gets multiplied by the numerator and the denominator by the denominator. The numerators and denominators stay separate, so we can take the square root separately. So let's complete our problem, don't forget the plus or minus in the answer.

$$x^2 = \frac{4}{9} \qquad \sqrt{x^2} = \sqrt{\frac{4}{9}} \qquad x = \pm\frac{2}{3}$$

Realize that we could have a problem where the answer will be irrational. In that case, we would either leave the answer as a plus or minus square root or approximate the solution.

$$x^2 = 50$$

$$\sqrt{x^2} = \sqrt{50}$$

$$x = \pm\sqrt{50} \approx \pm 7.1$$

Solving $x^3 = p$ where p is rational

If we used the square root to solve problems where the variable had been squared, what will need when the variable has been cubed?

$$x^3 = 27$$

If you thought we would need to take the cube root, you are correct. Since the cube root is the inverse operation to cubing, we will use that to isolate the variable as follows:

$$x^3 = 27$$

$$\sqrt[3]{x^3} = \sqrt[3]{27}$$

$$x = 3$$

Remember that we can take cube roots of negative numbers, unlike square roots. That means that the following problem is possible.

$$x^3 = -8$$

$$\sqrt[3]{x^3} = \sqrt[3]{-8}$$

$$x = -2$$

Notice that we could also take the cube root of a fraction in the same way we did with square roots.

$$x^3 = \frac{27}{64}$$

$$\sqrt[3]{x^3} = \sqrt[3]{\frac{27}{64}}$$

$$x = \frac{3}{4}$$

We will limit problems of this nature to ones where we can actually find the cube root.

Lesson 5.5

Solve.

1. $x^2 = 100$
2. $x^2 = 196$
3. $x^2 = 25$
4. $x^2 = 1$
5. $x^2 = 81$
6. $x^3 = 1$
7. $x^3 = 64$
8. $x^3 = -27$
9. $x^3 = -64$
10. $x^3 = -1$
11. $x^2 = \frac{25}{36}$
12. $x^2 = \frac{49}{16}$
13. $x^2 = \frac{64}{81}$
14. $x^3 = -\frac{27}{64}$
15. $x^3 = \frac{1}{8}$
16. $x^2 = 64$
17. $x^2 = 49$
18. $x^2 = 144$
19. $x^3 = -8$
20. $x^3 = 1000$
21. $x^3 = -125$
22. $x^2 = \frac{100}{121}$
23. $x^2 = \frac{4}{36}$
24. $x^3 = \frac{1}{125}$
25. $x^3 = 0.125$
26. $x^2 + 25 = 50$
27. $x^2 - 25 = 0$
28. $x^2 - 16 = 10$
29. $x^2 + 13 = 36$
30. $x^2 = 200$
31. $x^3 + 5 = 13$
32. $x^3 + 1 = 28$
33. $x^3 - 2 = 62$
34. $x^3 - 10 = 115$
35. $x^3 = \frac{8}{27}$